Problem solving
A CPD pack to support the learning and teaching of mathematical problem solving
Acknowledgement

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Introduction

The materials in this pack need to be read in conjunction with Excellence and Enjoyment: learning and teaching in the primary years. Introductory guide: continuing professional development (DfES 0243-2004 G).

This guidance is the first in a series of materials designed to help all staff involved in the teaching of mathematics to:
- identify problems of a particular type and the strategies that children can use to solve them;
- construct teaching sequences for teaching problem solving;
- incorporate problem solving within the mathematics curriculum;
- develop children’s reasoning and explanation skills;
- engage in whole-staff discussion on the above.

Proposed model for use of these materials in schools

These materials are designed to support the staff development programme in a primary school. They include guidance for an initial staff meeting and a follow-up meeting (supported by video sequences), together with pairs of lessons for each age group to Year 6 (including the Foundation Stage) for you to use with your own classes. The lessons follow a common structure; they include features such as reviews to discuss strategies and introduce questions to extend the challenge of the problem. In the follow-up meeting you and your staff will meet together to discuss how the lessons went, the children’s responses, achievements and progression. You could also discuss the guidance on producing similar problems, and examples of how the strategies used in the lessons could be applied to solving problems in other areas of the curriculum.

There are some decisions you will need to take before using these materials for a staff meeting. You will need to decide which of the video sequences to use with the staff. In the notes you will find that there are points to make for any of the sequences you use and points that are specific to each sequence. Considering these points, when watching the videos, will help you to decide which sequences to use. You might want to draw to the attention of appropriate members of staff the sequences that you do not use so that they can choose to watch them at some other time if they see fit.

You will need to tailor the material in the second staff meeting to the needs of your staff. You may feel that this meeting is too full and that to cover everything in sufficient detail it is necessary to hold the work on children’s responses over to another meeting.

Background to the materials

There are two aspects to teaching problem solving:
- teaching specific strategies to solve particular types of problem, for example in units on reasoning about number or shape;
- posing questions in ‘everyday’ teaching for children to practise and develop their general mathematical thinking and reasoning skills across the full mathematics curriculum, not just in the units on problem solving.

These materials will focus on the first aspect.
Classification of problems

It can be helpful to classify problems to help you to sort problems and to identify the problem-solving strategies that you need to teach. This will help you to ensure that children experience a variety of problems and are taught a range of strategies needed to solve them. It is also useful to distinguish between the children’s problem-solving strategies and the teaching approach – what the teacher does to help the children acquire the problem-solving strategy. These materials use the following classification for problems:

- Finding all possibilities;
- Logic problems;
- Finding rules and describing patterns;
- Diagram problems and visual puzzles;
- Word problems.

National Numeracy Strategy materials focusing on word problems have already been produced – Solving word problems in National Numeracy Strategy professional development materials 3 and 4 (NNS 65) – so these materials will focus on the other four types of problem. Problems do not always fall neatly into the five categories given. However, the classification helps to flag the strategies that children might draw upon and use to solve them, and the learning approaches you can plan to teach.

Finding all possibilities

The problem below could be described as a ‘Finding all possibilities’ problem, though it is also a word or story problem.

**Spaceship**

Some Tripods and Bipods flew from planet Zeno. There were at least two of each of them.

Tripods have 3 legs.
Bipods have 2 legs.
There were 23 legs altogether.

How many Tripods were there?
How many Bipods?
Find two different answers.

Teaching objectives
Solve mathematical problems or puzzles.
Count on in steps of 2 or 3.
Know multiplication facts for 2 and 3 times tables.

Taken from page 45 of Mathematical challenges for able pupils in Key Stages 1 and 2.
This problem is not simply asking the children to identify the calculations they need and then carry them out (as we usually ask them to do for word problems). The problem is asking the children to sort out from the information they are given what combinations of 2s and 3s they can make to meet the condition (23 legs). To do this systematically children might first produce tables or lists of multiples of 2s and 3s that they can then scan to find a total of 23. The children’s strategies might include generating, listing and organising or recording data.

To help children tackle this type of problem, the teaching approach might start with activities that generate lists. Introducing simple questions such as ‘What is the smallest number that appears in both the 4 and the 6 times table?’ will encourage children to list the tables. Giving children similar problems that involve listing will develop these skills and promote the strategies.

A useful teaching approach is to transform the problem by changing the context and the information while keeping the problem type the same. In this case, the context could involve 3- or 4-legged stools or chairs and a total of 35 legs; or packets of chocolate bars holding 4 and 6 bars and a total of 38 bars.

**Logic puzzles**

This problem gives the children just enough information. No calculations are involved, but the challenge is to hold one piece of information while interpreting another. To solve this puzzle children will need to look at the four statements and decide which they would start with to help them to identify some of the shapes’ colours. This prioritising and comparison of statements is a skill that children need to develop to solve this type of problem. As they progress through the clues, they also need to see the effect that each statement has on the position of the other colours.
Finding rules and describing patterns

One block is needed to make an up-and-down staircase, with one step up and one step down.

4 blocks make an up-and-down staircase with 2 steps up and 2 steps down.

How many blocks would be needed to build an up-and-down staircase with 5 steps up and 5 steps down?

This example is taken from the NRICH website: http://nrich.maths.org. (NRICH is a website managed by the University of Cambridge, which provides a range of activities and problems for use in primary schools)

Teachers may be quite familiar with the way in which these problems are used to develop reasoning. However, it may be that children are not familiar with all the strategies needed, and may not draw on them to solve problems. Children may start with a list and from the list make some observations about the bricks needed to build the staircase, and the pattern of numbers in the list. They would use these observations to inform their strategies, which would include describing a pattern, finding the next item in a pattern and generating a rule for the pattern.

Diagram problems and visual puzzles

Spot the shapes 1

1. How many triangles can you count?

2. How many rectangles can you count?

3. Draw your own diagram to count triangles. How many can a friend find? Can you find more?

Teaching objectives
- Solve mathematical problems or puzzles
- Visualise 2-D shapes
- Explain methods and reasoning

Taken from page 40 of Mathematical challenges for able pupils in Key Stages 1 and 2
With this problem, being systematic is important. The children need to keep track of the triangles and rectangles they identify to avoid any double counting. Annotating the diagram will help to achieve this. They need to decide on a strategy to ensure that they find all the shapes. Starting with the top two triangles and then increasing the size would be one such strategy.

**Teaching the strategies to solve these problems**

A focus on one or two types of problem should be sufficient for a term, building on this and introducing another type of problem during the following term. Over a week, the children learn how to apply the strategies they have been taught in a particular lesson to similar problems in the following lessons.

For these types of problem some of the teaching approaches include those listed on the back of handout 2, *Types of problem and appropriate strategies*.

Children need to be taught the strategies and to be shown how they can apply these systematically to problem solving. For many children the hit-and-miss approach they use when gathering information and their poor management of information limits their ability to work systematically. Being systematic is about first deciding what is being or can be asked, what information can be used and what data can be generated. Children then need to identify a way that helps to control the questioning and identify from the data gathering which key information must be selected. Such an approach requires practice of taught skills and strategies. Simply giving children problems is not enough.

This set of materials looks in more detail at one type of problem, ‘Finding all possibilities’, the strategies for solving this type of problem, and teaching approaches to enable children to develop these strategies.

These materials aim to help teachers to think about what strategies they can teach children. They also use questioning to develop children’s reasoning and communication skills.

The materials include:
- examples of the ‘Finding all possibilities’ type of problem, the strategies used to solve them and ideas for creating similar problems;
- teaching sequences (pairs of lessons) exemplifying different teaching approaches that allow for different learning styles and develop a range of problem-solving strategies;
- video sequences which will illustrate teaching approaches to problem solving.

**Next steps**

The intention is to evaluate the usefulness and impact of these materials to determine whether we produce further materials illustrating other types of problem, strategies and teaching approaches and what form these materials might take. We also plan to consider how we might develop the theme of improving children’s reasoning and communication skills across the mathematics curriculum, focusing on questions that encourage children’s mathematical thinking.
Notes to guide whole staff – discussion 1

Outline of the staff meeting

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<th>Introduction</th>
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<td>1. Introduction to classification of problems</td>
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Objectives:
- To present staff with the background to the work in problem solving
- To introduce staff to the lesson materials
- To identify lessons that staff will teach to inform subsequent meetings
- To agree as a staff some principles for developing problem solving

Key messages:
- It is important that children are introduced to a range of problems and taught the associated problem-solving strategies.
- Children need to be helped to develop ways of working systematically and should become more independent problem solvers as they move through the school.
- The lessons presented are for guidance. They may need to be adapted to the needs of the class.

**Introduction**

Share the objectives for the session.

Explain that this guidance is the first in a possible series of support materials to help teachers to:
- recognise problems of a particular type and the strategies used to solve them;
- develop teaching sequences for teaching problem solving;
- incorporate problem solving within the mathematics curriculum;
- develop children’s reasoning and explanation skills;
- identify, share and develop key principles and effective teaching approaches that enable children to be confident problem solvers.

1. **Introduction to classification of problems**

Explain that there are two different aspects of problem solving. The first is the problem-solving lesson. These materials will look at ways to classify problems and the skills necessary for solving them. An important element of this work will be the teaching of these skills to equip children to be able to have a range of strategies to tackle problems.

The second aspect will be the place of problem solving in everyday lessons. This takes as a focus how a teacher by way of questioning and prepared activity can ensure that a substantial proportion of lessons have an element of problem solving.
Say that it can be helpful to classify problems to help teachers to sort problems and to identify the strategies they need to teach, and for children to recognise problems and the strategies they need to solve them. There are various ways of classifying problems. What is important is to ensure that children experience a variety of problems and are taught the range of strategies necessary to solve them. It is also useful to distinguish between the children’s problem-solving strategies and the teaching approach – what the teacher does to help the children acquire the problem-solving strategy.

These materials use the following classification:

- Finding all possibilities;
- Logic problems;
- Finding rules and describing patterns;
- Diagram problems and visual puzzles;
- Word problems.

National Numeracy Strategy materials focusing on word problems have already been produced, so these materials will focus on the other four types of problem.

2. Finding all possibilities

Look at handout 1, which gives the spaceship problem (or refer to page 45 of Mathematical challenges for able pupils in Key Stages 1 and 2). Say that this could be described as a ‘Finding all possibilities’ problem, though it is also a word or story problem.

Say that the problem is not asking the children to identify any calculations they need and then carry these out (as we usually ask them to do for word problems). The problem is asking the children to sort out, from the information they are given, what combinations of 2s and 3s they can make to meet the condition (23 legs). Ask teachers to have a go at this problem and discuss the methods they have used.

Q. What strategies did they use?
Q. Did they produce any lists or tables?
Q. What would they agree is a systematic approach to solving this problem?

To help children tackle this type of problem, the teaching approach might include activities that generate lists. Introducing simple questions such as “What is the smallest number that appears both in the 4 and the 6 times table?” will encourage children to list the tables. Emphasise that giving children similar problems that involve listing will develop these skills and promote the strategies.

A useful teaching approach is to transform the problem by changing the context and the information while keeping the problem type the same. In this case, the context could involve 3- or 4-legged stools or chairs and 35 legs. Ask the teachers to offer other contexts that keep the problem type the same.

Briefly collect examples that show how quickly such a range of similar problems can be generated. Agree that this involved changing the objects, the number of shared characteristics per object and the total to be made.

Say that for these and other types of problem, some of the teaching approaches include those listed on the back of handout 2, Types of problem and appropriate strategies.
Emphasise to the teachers that children need to be taught the strategies and to be shown how they can apply these systematically. For many children the initial problem is their hit-and-miss approach when gathering information and poor management of information. Being systematic is about first deciding what can be asked or what data can be generated and then identifying a way that helps to control the questioning or data gathering from which the key information can be selected. Such an approach requires practice of taught skills and strategies. Simply giving children problems is not enough. These materials aim to help teachers teach the necessary strategies.

3. Introduction to teaching materials

Explain that these materials look at a group of problems that fall into this same category ‘Finding all possibilities’. It is intended that the whole school will use these materials to focus on this aspect of problem solving. There are two lessons for each age group (and a selection of activities for the Foundation Stage). This ensures that everyone has similarly structured lessons that will provide a common focus for the second staff meeting. Talk through the following features of the lessons.

- Progression has been built across the two lessons; the work in the second lesson builds on the work of the first. Progression has been built across the years. Staff may need to check that the children in their class have the appropriate prerequisite knowledge and skills to access the lessons in full.
- The lessons are set out with the main ‘story’ of the lesson in the middle column. Down the left-hand side are suggestions for how the lesson might be modified. On the right-hand side there are illustrative thoughts behind the planning of the lessons and suggestions on aspects for feedback into the following staff meeting.

Stress that it is important not to use the lesson as a script, but to adapt the lesson to the needs of the children. The comments in the right and left columns are designed to help with this task, while preserving the key feature of problem solving to be discussed at the next staff meeting.

Allow staff time to read through and discuss the teaching materials for their age group. Ask them to identify how the children are taught to generate the necessary information and to prioritise the information. Draw attention to how the children solve two similar problems in order to practise the strategy of listing all possibilities using structured and systematic methods.

4. Looking at examples of lessons

Show the selected video sequences taken from lessons where teachers are using these teaching sequences. Ask staff to focus on:
- how the problem was introduced to the class;
- the teaching approaches used by the teacher to develop the children’s problem-solving skills;
- the responses of children to the tasks set and the strategies they used.

Take feedback.
In the discussion highlight the positive features that staff can draw upon and build into their own teaching.

The following points should be drawn from the videos.

- The teachers found ways for the children to engage with and explore the problem. These are different in each year group. In Year 1, the teacher introduces a simpler problem to start with (4p) and supports the children in establishing one solution before finding all solutions to the 6p problem. In Year 4, a similar problem is set for the whole class to work through. The strategies used for solving this problem are discussed before the main problem is introduced. In Year 6, the teacher engages the children with different aspects of the problem before encouraging them to solve it.
- The teachers demonstrate some of the processes that the children will find useful. These are intended to promote systematic working.
- The children are encouraged to record their work systematically, building tables and lists.
- Children are working in pairs or groups to encourage discussion and to allow them to rehearse arguments before sharing these with the class.
- The lessons are structured to allow for mini-plenaries so that ideas can be discussed and children can build on each other’s suggestions and approaches.
- Children’s work is used to extend the discussion. This requires the teacher to have identified some work to share with the rest of the class.

Specific points for the Year 1 video

- The scene is set with practical and visual resources.
- The teacher comments on the things she has observed, making clear what she values in the way the children are working.
- Positive use is made of a child’s error (3p + 1p).
- The teacher decides which group to focus on during the main part of the lesson.
- There is an extensive use of whiteboards, which helps children to feel comfortable in experimenting with some of their ideas.
- The teacher stresses the use of checking.
- The teacher models the use of a number sentence, linking it to the use of coins.
- The teacher ensures that there is a focus on the use of the correct vocabulary.

Specific points for the Year 4 video

- The teacher encourages the children to revisit the previous day’s work to emphasise the problem-solving strategies they have been using.
- Children’s solutions are used to illustrate the teaching points the teacher wants to make.
- The teaching assistant works with the same problem, but provides children with an alternative, practical representation to help them to access the problem.
- The children rehearse answers in pairs before sharing them with the whole class.
- The teacher uses a different problem to encourage children to evaluate their previous strategies. (It is now better to use the largest number rather than the smallest number.)
- The teacher uses a familiar context to introduce the problem.

Specific points for the Year 6 video

- ICT has been used to ensure that the lesson keeps a good pace. The ICT allows the children to reorganise and check their work.
5. What next?

Agree with staff on when they might each teach the relevant two lessons with their class, and an appropriate point at which the future staff meeting might be held to discuss the progress of the children.

Remind the teachers that the plans are for guidance only and should be adapted to meet the needs of their own class.

Refer to pages 5, 6 and 7 in Excellence and Enjoyment: learning and teaching in the primary years. Introductory guide: continuing professional development (DfES 0243-2004 G). With the staff decide which of the features from this document to use. For example, some teachers might be able to work with a colleague and observe one another’s lessons, some might choose to meet between the two lessons to review the progress made and make adaptions accordingly, some might suggest different ways to use peer support, others might have access to external support such as a numeracy consultant.

Explain that the follow-up staff meeting will provide an opportunity for staff to discuss and agree on what has worked well and use the discussion to draw up some principles of good practice to guide the teaching of problem solving. As everyone will have taught two lessons that focus on problems of a similar type – ‘Finding all possibilities’ – this will provide the opportunity for the whole staff to focus on progression in problem solving. It will be interesting to find out how children in different year groups tackled the problems and when and how they were systematic in their approach. Children in Years 2 and 5, say, might both be creating lists to help them identify different possibilities, but are the same strategies used in each of these years? Or are there some evident differences between the strategies used? Say that as staff you are engaging in some action research to help answer these questions, which will enable them to move forward. This is on the underlying principle of the CPD model set out in Excellence and Enjoyment: learning and teaching in the primary years. Introductory guide: continuing professional development (DfES 0243-2004 G).

To help with these tasks, before the next staff meeting each teacher is to provide examples of work undertaken by children in the low-, middle- and upper-ability groups.

Say that you also want staff to be ready to discuss the following.

- What did we do to help the children gain a better understanding of the problem?
- How did we teach the children to be more systematic?
- What are the key features of teaching and learning when problem solving that we should incorporate into our lessons?
- What barriers did we identify that might have stopped the children being successful?
Notes to guide whole staff – discussion 2

Outline of staff meeting

1. Introduction

Explain that this staff meeting is to review the problem-solving activities used by staff and to decide on any further steps that need to be taken. Introduce the objectives.

2. Teaching strategies

Ask staff, in groups, to discuss the following, with reference to age-specific lessons/activities.

- What did we do to help the children gain a better understanding of the problem?
- How did we teach the children to be more systematic?
- What are the key features of teaching and learning when problem solving that we should incorporate into our lessons?

Encourage each group to record on a sheet of paper their responses to these questions. Bring the groups together and use their recorded responses to agree principles of good practice when teaching problem solving.

The principles that might be included are:

- introducing the problem in a way that engages the children through the use of practical materials or by setting contexts that are pertinent to the children;
- allowing children to start by reading and exploring the problem in groups to ensure that they have an understanding of what is required and an idea as to how they might tackle the problem;
- discussing the strategies the children might use and reviewing what they have used during the lesson;
- finding examples of children’s strategies to share with and compare against those used by other children in the class;
● explaining to the children how their strategies help them to become more systematic;
● asking questions that encourage children to explain their thinking;
● encouraging children to look at each other’s work, discuss methods and explain solutions;
● asking children to ask each other questions about the problem to check that they share the same understanding;
● working through a similar problem together using the same strategies as before.

Say that the agreed features will be typed up and can be used as a checklist when planning or observing teaching.

Ask the groups to identify any barriers that prevented the children from being successful.

Take feedback.

Possible barriers for children could be:
● unfamiliarity with this type of problem;
● relying on trial and error and hit-and-miss strategies;
● thinking that there was only one solution;
● not reading the problem carefully enough;
● not recording carefully enough the data they generated;
● not talking about the problem with other children;
● not annotating the problem sheet;
● not being secure enough with the strategies.

Decide how the barriers might be tackled by the principles identified in the previous discussion.

Decide how the principles can be incorporated into future planning and teaching, and begin to map out how what is taught in Reception and Year 1 can be built on in Year 2 and Year 3, etc.

3. Children’s responses

Ask the staff to share any responses from children that surprised them – both the positive and negative responses. Discuss the factors that might have led to or influenced these responses. Identify any factors that lead to children being insecure when problem solving and those factors that stimulate the children’s interest, enthusiasm and willingness to stick at the task.

Organise the staff into groups who teach different ages. Give out handout 3.

Explain that handout 3 lists seven strategies that children could use when solving “Finding all possibilities” problems. Each group of staff is to look at the response to the problem from children across the different age groups. They are to note the year group of the child and look for examples of how the children are using the strategy and then describe this.

Make sure that each group looks at the responses of some able and some less able children. After a while say that you also want the groups to:
look for evidence of children using strategies they were taught in the first lesson to solve problems in the second lesson;
look for evidence of progression in the problem-solving strategies used by children across the age range;
select examples of children’s work and as a group decide what questions they would ask these children to help them develop their future strategies and be more systematic.

Draw the whole group together and discuss the evidence and descriptions of strategies. If appropriate, add any new principles of good problem solving to those agreed earlier. Use the responses to begin to identify and record a list of the key steps in progression in problem solving across the year groups. Draw out the details in terms of the different strategies each year group might be introduced to and asked to build on, and the depth of understanding the children in each year group need to acquire to become more proficient and systematic problem solvers.

4. What next?

Using the agreed principles of good practice identify those aspects that the staff agree are taught well throughout the school and those aspects that need to be developed.

Finish the meeting by explaining that the two lists started in the meeting are to be developed. The principles should inform planning and teaching; the progression in children’s strategies should inform learning and expectations. Agree the actions that each member of staff will take to address those aspects that need further development.

These actions could include:
revise planning to ensure that there are opportunities for children to develop the relevant problem-solving strategies;
developing and sharing problems that are of a particular type;
supporting each other in planning problem-solving lessons;
observing each other when teaching problem solving;
reviewing children’s work against the progressed strategies.
Spaceship

Some Tripods and Bipods flew from planet Zeno. There were at least two of each of them.

Tripods have 3 legs.
Bipods have 2 legs.
There were 23 legs altogether.

How many Tripods were there?
How many Bipods?

Find two different answers.

Teaching objectives
Solve mathematical problems or puzzles.
Count on in steps of 2 or 3.
Know multiplication facts for 2 and 3 times tables.

Taken from page 45 Mathematical challenges for able pupils in Key Stages 1 and 2
Handout 2

Types of problem and appropriate strategies

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<tr>
<th>Finding all possibilities</th>
<th>Logic puzzles</th>
<th>Finding rules and describing patterns</th>
<th>Diagram problems and visual puzzles</th>
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<tbody>
<tr>
<td>● Have a system for finding the possibilities, e.g. start with the smallest number</td>
<td>● Identify the given facts and prioritise them</td>
<td>● Decide on the information you need to describe and continue the pattern</td>
<td>● Identify the given information and represent it in another way</td>
</tr>
<tr>
<td>● Organise the recording of possibilities, e.g. in an ordered list or table</td>
<td>● Look for any relationships and patterns in the information given</td>
<td>● Give examples to match a given statement and ones which do not</td>
<td>● Use a systematic approach to solve the problem and a way of recording if necessary</td>
</tr>
<tr>
<td>● Use a method of tracking what has been included and what has not</td>
<td>● Use one piece of information at a time and see what effect it has, then keep one fixed and test the other</td>
<td>● Describe a rule of a pattern or relationship in words or pictures</td>
<td>● Use drawings or annotations to help visualise the problem using familiar shapes or patterns</td>
</tr>
<tr>
<td>● Have a way of deciding when all possibilities have been found</td>
<td>● Choose and use a recording system to organise the given information</td>
<td>● Predict the next few terms in a sequence to test the rule</td>
<td>● Try other possibilities to check the solution</td>
</tr>
<tr>
<td></td>
<td>● Check that the answer meets all the criteria</td>
<td>● Use a rule to decide whether a given number will be in the sequence or not</td>
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</tbody>
</table>

Tackling a problem or puzzle

read the given information and identify the question \rightarrow decide on where to start \rightarrow be systematic in approach and recording \rightarrow review and decide how to continue \rightarrow check that the answer works
Teaching approaches include:

- Presenting problems with too little or too much information, requiring the children to decide what extra they need or what they can discard;

- Setting problems that allow the children to represent solutions in different ways, with practical aspects, pictures, diagrams, notation, symbols, etc.;

- Using the same problems more than once to demonstrate alternative problem-solving strategies that the children can use;

- Comparing problems of different types to demonstrate why one approach is more effective than another;

- Giving children a bit of information and a context for them to construct and present a problem of a type similar to the ones they have been solving;

- Offering children four problems, say, with two of the same type that the children must identify and solve using the same strategies;

- Presenting children with alternative strategies and solutions that help them to make decisions about how to start a problem;

- Coaching children on how to present their solutions during the lesson, using a common vocabulary that everyone understands, such as listing, sorting, reordering;

- Giving children time to practise a strategy by solving part of a problem or starting half-way in and then working backwards to get a full solution.
## ‘Finding all possibilities’ type of problem

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<tr>
<th>Strategies</th>
<th>Description of examples from children’s work</th>
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<tr>
<td>● Children have a starting strategy to begin to explore the problem</td>
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<tr>
<td>● Children have a system for recording the possibilities, e.g. pictures, letters, symbols, etc.</td>
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<tr>
<td>● Children organise the data collected in a list or table that helps to identify any gaps</td>
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<tr>
<td>● Children have a system for recording the data to identify all possibilities, e.g. start with the smallest, keep one thing fixed while changing the others</td>
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<tr>
<td>● Children check for any repeats and remove them</td>
<td></td>
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<tr>
<td>● Children have a system for tracking what has been included and what has not</td>
<td></td>
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<tr>
<td>● Children have a way of deciding if and when all possibilities have been found</td>
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